

Ultracold Fermion Cooling Cycle using Heteronuclear Feshbach Resonances

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We consider an ideal gas of Bose and Fermi atoms in a harmonic trap, with a Feshbach resonance in the interspecies atomic scattering that can lead to formation of fermionic molecules. We map out the phase diagram for this three-component mixture in chemical and thermal equilibrium. Considering adiabatic association and dissociation of the molecules, we identify a possible cooling cycle, which in ideal circumstances can yield an exponential increase of the phase-space density.

Introduction—Feshbach resonances [1] have become a powerful tool for controlling atomic interactions in ultracold gases [2, 3, 4, 5, 6, 7, 8, 9]. By tuning an external magnetic field, the energy of a molecular bound state can be adjusted to be on resonance with a scattering state of two free atoms, thus facilitating external control of the atomic interactions. Tuning the molecular state below threshold can lead to the production of stable ultracold diatomic molecules. This has been accomplished both in bosonic [6] and fermionic gases [7]. In all of these cases the diatomic molecules are homonuclear bosons. Two recent experiments, at MIT [8] and at JILA [9] have observed Feshbach resonances in Bose-Fermi atomic mixtures. The molecular species in this case is a heteronuclear fermion.

In this work we construct the phase diagram for an ideal trapped gas of bosonic and fermionic atoms, and the heteronuclear dimer formed from them [10]. In contrast to previous work on Fermi gases [11, 12], we find that adiabatic passage from free atoms to molecules can lead to cooling of the system under conditions comparable to those of current experiments. Based on this observation we propose a thermodynamic cooling cycle, consisting of adiabatic interconversion between atoms and molecules in conjunction with selective removal of atoms. This cycle is capable of reducing the absolute temperature of the system while increasing the atomic phase-space density.

Equilibrium theory—We consider an ideal gas mixture composed of fermionic (f) and bosonic (b) atoms and heteronuclear fermionic molecules (bf) trapped in an anisotropic harmonic optical trap. The trapping frequencies for the different components are taken to be:

$$\omega_f^i = \sqrt{\frac{\alpha_f^i}{m_f}}, \quad \omega_b^i = \sqrt{\frac{\alpha_b^i}{m_b}}, \quad \omega_{bf}^i = \sqrt{\frac{\alpha_f^i + \alpha_b^i}{m_f + m_b}}, \quad (1)$$

where $i = \{x, y, z\}$, α_k^i is proportional to the atomic polarizability, and m_k are the particle masses, with $k = \{f, b, bf\}$. The expression for the molecular frequency ω_{bf}^i in terms of the atomic frequencies is appropriate when the molecular internuclear separation is

large compared to the mean atomic radii. It is possible to change the ratio ω_f^i/ω_b^i over a wide range by varying the wavelength of the trapping laser, and Ref. [13] gives specific prescriptions for doing this. We assume chemical and thermal equilibrium: $\mu_f + \mu_b = \mu_{bf}$ and $T_f = T_b = T_{bf} = T$, where μ_k are the chemical potentials of the three components, and T is the temperature. The energy of the molecular state as measured from the atomic dissociation continuum is ϵ_{res} , which is a function of an applied magnetic field. The results of this paper are based on considering the phase diagram of this three-component system as a function of T , ϵ_{res} , and the atomic populations.

We employ a semi-classical approximation, such that populations of each component are given by

$$N_f(T, \mu_f) = \left(\frac{k_B T}{\hbar \bar{\omega}_f}\right)^3 \mathcal{F}_3(z_f), \quad (2)$$

$$\tilde{N}_b(T, \mu_b) = \left(\frac{k_B T}{\hbar \bar{\omega}_b}\right)^3 \mathcal{G}_3(z_b), \quad (3)$$

$$N_{bf}(T, \mu_{bf}, \epsilon_{\text{res}}) = \left(\frac{k_B T}{\hbar \bar{\omega}_{bf}}\right)^3 \mathcal{F}_3(z_{bf}), \quad (4)$$

where the fugacities z_k are

$$z_f = e^{\mu_f/k_B T}, \quad z_b = e^{\mu_b/k_B T}, \quad z_{bf} = e^{(\mu_{bf} - \epsilon_{\text{res}})/k_B T}. \quad (5)$$

Here k_B is Boltzmann's constant, and $\mathcal{F}_n(z)$ and $\mathcal{G}_n(z)$ are the Fermi-Dirac and Bose-Einstein integrals, respectively [12, 14]. This approximation is valid provided $k_B T \gg \hbar \bar{\omega}_k$, where $\bar{\omega}_k = (\omega_k^x \omega_k^y \omega_k^z)^{1/3}$ is the mean harmonic frequency. $\tilde{N}_b(T, \mu_b)$ represents the number of non-condensed Bose atoms; we denote the number of Bose condensed atoms by N_c , and $N_b = N_c + \tilde{N}_b$.

For a given T , ϵ_{res} , total number of atoms N , and boson-fermion population difference ΔN the atomic chemical potentials are determined by the conditions

$$N = N_f(T, \mu_f) + N_b(T, \mu_b) + 2N_{bf}(T, \mu_{bf}, \epsilon_{\text{res}}), \quad (6)$$

$$\Delta N = N_f(T, \mu_f) - N_b(T, \mu_b). \quad (7)$$

For T above a critical temperature T_c all bosons are thermal and $N_c = 0$, whereas for $T < T_c$ a Bose-Einstein condensate (BEC) emerges: $N_c > 0$, $\mu_b = 0$ ($\mu_{bf} = \mu_f$), and

$$\tilde{N}_b(T, \mu_b = 0) = \zeta(3) \left(\frac{k_B T}{\hbar \bar{\omega}_b}\right)^3, \quad (8)$$

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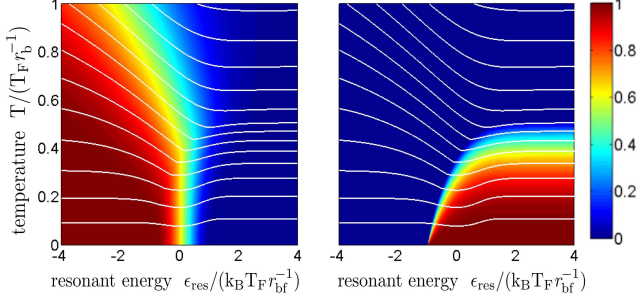


FIG. 1: Phase diagram for trapping parameters of the JILA system [15], with $\Delta N = 0$; this representation is universal in N subject to validity of the semi-classical approximation. Left frame: molecule fraction, η_{bf} , as a function of ϵ_{res} and T ; right frame: condensate fraction, η_c . White lines indicate adiabatic trajectories, i.e. curves of constant entropy.

where $\zeta(n)$ is the Riemann zeta function.

Phase diagrams—We now discuss the phase diagrams resulting from the numerical solutions to Eqs. (6) and (7). Defining the molecular and condensate fraction as $\eta_{bf} \equiv 2N_{bf}/N$ and $\eta_c \equiv N_c/N_b$, respectively, we plot these quantities in Fig. 1 versus temperature and ϵ_{res} for a system where $\Delta N = 0$, using trapping parameters for the JILA experiment [15]. The T and ϵ_{res} axes are normalized by T_F/r_b and $k_B T_F/r_{bf}$, respectively, where T_F is the Fermi temperature of the fermionic atoms, $k_B T_F = \hbar \bar{\omega}_f (6xN)^{1/3}$, $x = (N_f + N_{bf})/N$ is the total fermionic fraction, $r_{bf} = \bar{\omega}_f/\bar{\omega}_{bf}$ and $r_b = \bar{\omega}_f/\bar{\omega}_b$. For a finite population difference the maximum number of molecules is $N_{bf}^{\max} = (N - |\Delta N|)/2$.

When $\Delta N < 0$, there will always be free bosons in the system, and condensation is possible for any value of ϵ_{res} . When $\Delta N \geq 0$, there is a critical resonance energy, $\epsilon_{res}^{(c)}$, at which the condensate fraction vanishes at $T = 0$. This can be seen in Fig. 1. Setting $N_b = 0$ in Eqs. (6) and (7) and $\mu_{bf} = \epsilon_{res}^{(c)} + (6N_{bf})^{1/3} \hbar \bar{\omega}_{bf}$ at $T = 0$ we find that

$$\frac{\epsilon_{res}^{(c)}}{k_B T_F} = \left(2 - \frac{1}{x}\right)^{1/3} - \frac{1}{r_{bf}} \left(\frac{1}{x} - 1\right)^{1/3}. \quad (9)$$

In a similar manner we can solve for the critical temperature T_c for BEC in the limit $\epsilon_{res}/k_B T_F \gg 1$, where $N_{bf} = 0$. From Eq. (8) we find

$$\frac{T_c}{T_F} = [6\zeta(3)]^{-1/3} \frac{1}{r_b} \left(\frac{1}{x} - 1\right)^{1/3}, \quad (10)$$

In the case of equal populations $x = 1/2$, the limits take a simple form $\epsilon_{res}^{(c)} = -k_B T_F/r_{bf}$ and $T_c \simeq 0.518 T_F/r_b$. These two points define the left and top boundaries of the condensate region in Fig. 1.

Adiabatic sweep—During an adiabatic sweep, the resonant energy is varied on a time scale much longer than the average relaxation time of the system [16]. Then

entropy is conserved, and the system follows an adiabatic path through the phase diagram. The total entropy of the system, $S(T)$, is given by the sum of the entropies of each of the three components:

$$S_f(T, \mu_f) = k_B N_f \left[4 \frac{\mathcal{F}_4(z_f)}{\mathcal{F}_3(z_f)} - \frac{\mu_f}{k_B T} \right], \quad (11)$$

$$S_b(T, \mu_b) = k_B \tilde{N}_b \left[4 \frac{\mathcal{G}_4(z_b)}{\mathcal{G}_3(z_b)} - \frac{\mu_b}{k_B T} \right], \quad (12)$$

$$S_{bf}(T, \mu_{bf}, \epsilon_{res}) = k_B N_{bf} \left[4 \frac{\mathcal{F}_4(z_{bf})}{\mathcal{F}_3(z_{bf})} - \frac{\mu_{bf} - \epsilon_{res}}{k_B T} \right] \quad (13)$$

The final temperature and chemical potentials at the end of the sweep are determined from the condition $S(T^i) = S(T^f)$ together with the constraints (6) and (7). Figure 1 shows the contours of constant entropy in the phase diagrams for the case of equal populations. Starting with a pure atomic system at $\epsilon_{res} \gg k_B T_F$ we observe that there is a temperature, T_{iso} , such that for $T^i > T_{iso}$ the final temperature after complete association of the gas is higher than the initial temperature. However, for $T^i < T_{iso}$ we find the interesting result that the temperature of the system decreases after the atoms have been converted into molecules. This can be understood from the fact that at low temperatures, the majority of the free bosons are condensed and do not contribute to S . By sweeping the value of ϵ_{res} across the resonance we go from an almost perfect Fermi sea of atoms to an almost perfect Fermi sea of molecules. But since $\bar{\omega}_{bf} < \bar{\omega}_f$, the final temperature must be lower if $S(T^i) = S(T^f)$.

An analytic expression for the relation between the initial and final temperatures can be obtained in high- and low- T limits. For $T \gg (T_F, T_c)$, the entropy of each component is of the Boltzmann form [14]

$$S_k(T) = k_B N_k \left\{ 4 - \ln \left[\left(\frac{\hbar \bar{\omega}_k}{k_B T} \right) N_k \right] \right\}. \quad (14)$$

One obtains T^f from $S(T^f) = S(T^i)$; in the case $x = 1/2$ the result is

$$\frac{T^f}{T_F} = 6^{1/3} e^{4/3} \left(\frac{\bar{\omega}_{bf}}{\bar{\omega}_b} \right) \left(\frac{T^i}{T_F} \right)^2. \quad (15)$$

The bosonic entropy for $T \ll (T_F, T_c)$ is:

$$S_b(T) = \frac{2\pi^4}{45} k_B \left(\frac{k_B T^i}{\hbar \bar{\omega}_b} \right)^3, \quad (16)$$

while for the fermionic atoms the low temperature expansion of $\mathcal{F}_n(z)$ [14] yields:

$$S(T) = \pi^2 k_B N_f \left(\frac{T}{T_F} \right) \left[1 - \frac{\pi^2}{5} \left(\frac{T}{T_F} \right)^2 \right]. \quad (17)$$

The entropy of the molecules is given by a similar expression. The adiabatic constraint $S(T^i) = S(T^f)$ in general

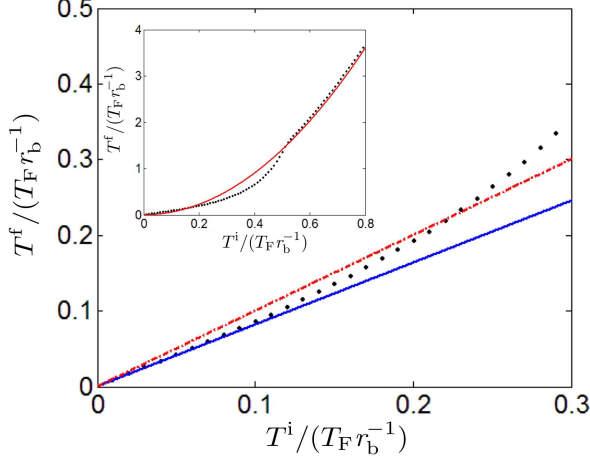


FIG. 2: Final temperature after adiabatic sweep from atomic to molecular configuration as a function of the initial temperature (dots). The frequency ratios correspond to those of the JILA experiment [15], and $x = 1/2$. The analytic relation (20) is shown for comparison (solid line), along with the line $T^f = T^i$ (dashed line). The inset shows the same numerical results compared with the analytic expression (15) in the high temperature limit (solid line).

does not lead to a simple compact expression. However, by setting $T^i = T^f \equiv T_{\text{iso}}$ we find the initial temperature for which an adiabatic sweep is also isothermal (in the sense that the temperature endpoints are equal):

$$\frac{T_{\text{iso}}}{T_F} = \frac{\sqrt{15}}{\pi x^{1/3}} \sqrt{\frac{r_{bf}(1-x)^{2/3} + (2x-1)^{2/3} - x^{2/3}}{4r_b^3 + 3r_{bf}^3}}. \quad (18)$$

If $T^i > T_{\text{iso}}$ the gas is heated by the adiabatic sweep, while a starting temperature $T^i < T_{\text{iso}}$ leads to cooling. Assuming extremely low temperatures, such that the entropy can be taken to be linear in T , a simple relation can be obtained for $\Delta N > 0$:

$$\frac{T^f}{T^i} = \frac{x^{2/3}}{(2x-1)^{2/3} + r_{bf}(1-x)^{2/3}}, \quad x \geq 0.5. \quad (19)$$

In the case of equal atomic populations, the relation becomes:

$$T^f = T^i / r_{bf}. \quad (20)$$

The same expression holds in the case of $\Delta N < 0$, where the ratio T^f/T^i is independent of $|\Delta N|$, as long as there is a condensate in both initial and final states. This is the regime where experiments are currently operating.

Figure 2 plots the dependence of the final temperature on the initial temperature for $x = 1/2$ for the JILA trap parameters [15] in comparison to the low- and high- T limits (20) and (15), respectively. For these parameters

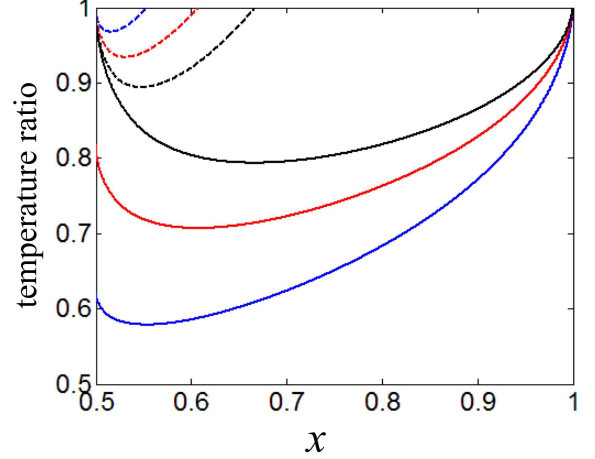


FIG. 3: Ratio T^f/T^i after adiabatic conversion of atoms to molecules (Eq. (19); solid lines), and ratio T^1/T_F^1 after one cooling cycle, steps 1-6, normalized by its initial value T^0/T_F^0 (Eq. (22); dashed lines), both as a function of the fermionic fraction x . The colors correspond to the JILA parameters [15] (red), the MIT parameters [17] (blue) and the best case $r_{bf} = 1$ (black).

Eq. (18) give $T_{\text{iso}} = 0.20T_F$, which is seen to provide a good estimate of the value of T^i for which $T^f = T^i$.

Cooling Cycle—We plot Eq.(19) in Fig. 3. As is evident, T^f/T^i depends strongly on the fermionic fraction x . In particular, there is an optimal fraction, $x_{\text{opt}} = [1 + r_{bf}^3]/[1 + 2r_{bf}^3]$, which maximizes cooling; it corresponds to the minimum of the T^f/T^i curve.

Since $x_{\text{opt}} > 1/2$ the following cooling cycle is possible:

1. Start with an atomic gas at $\epsilon_{\text{res}}/k_B T_F \gg 1$. Cool the gas to $T < T_{\text{iso}}$ through standard cooling techniques.
2. Alter the fermionic fraction to x_{opt} .
3. Sweep ϵ_{res} adiabatically to $-\epsilon_{\text{res}}/k_B T_F \gg 1$, forming the maximum number of molecules possible.
4. Remove the free atoms from the system.
5. Adiabatically increase ϵ_{res} into the dissociated regime.
6. Continue the cycle by repeating steps 2 to 5 until the desired final temperature is reached.

Assuming $T \ll T_F$ we can use (19) and (20) to calculate the final temperature of the system after n cycles:

$$T^n = T^0 \left(\frac{r_{bf} x^{2/3}}{(2x-1)^{2/3} + r_{bf}(1-x)^{2/3}} \right)^n, \quad (21)$$

where we have neglected a small decrease in temperature after suddenly decreasing the populations in steps 2 and 4. Notice that values of r_{bf} closer to unity produce better cooling; this can be attained by appropriate choice of wavelength of the trapping laser [13].

As is the case for evaporation, this cooling scheme leads to particle loss from the system in steps 2 and

4. This results in a reduction of the Fermi temperature T_F . Hence the figure of merit for increasing phase-space density is the decrease of the ratio of the temperature T to the Fermi temperature T_F at the end of the cycle. A simple calculation shows that after n cycles $T_F^n = T_F^0[(1-x)/x]^{n/3}$. The expression for T/T_F as a function of the number of cycles performed is then:

$$\frac{T^n}{T_F^n} = \frac{T^0}{T_F^0} \left[\frac{r_{bf}x}{(2x-1)^{2/3}(1-x)^{1/3} + r_{bf}(1-x)} \right]^n. \quad (22)$$

Fig. 3 shows this expression for $n = 1$ and Fig. 4 shows an example of the change in T/T_F , as a function of the number of cycles performed. For $x > x_{\text{opt}}$ the cycle results in a net increase of T/T_F , and for $x = x_{\text{opt}}$ the phase-space density remains unchanged. We find that there is a fermionic fraction, $\tilde{x}_{\text{opt}} < x_{\text{opt}}$, which maximizes the reduction of T/T_F ; this corresponds to the minimum of the respective curve depicted in Fig. 3. We plot it as a function of r_{bf} in the inset of Fig. 4.

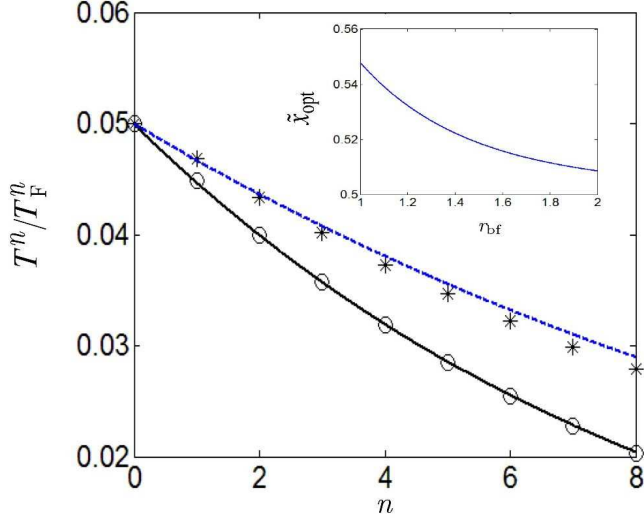


FIG. 4: Ratio of absolute to Fermi temperature after n cycles operated at $x = \tilde{x}_{\text{opt}}$. The numerical results (symbols) are compared with the analytical estimate (22), which neglects a small temperature decrease that occurs in steps 2 and 4 (lines). Results are shown for the JILA parameters [15] (asterisks and dashed line) and for the best case $r_{bf} = 1$ (circles and solid line). Inset: \tilde{x}_{opt} as a function of the frequency ratio r_{bf} .

The efficacy of the proposed cooling scheme is contingent on losses from three-body collisions being minimal. Presently, no estimates of their rates exist for the systems under consideration. Additionally, phase separation may become an issue, as the interspecies atomic scattering length diverges on resonance [18].

Conclusion—We have presented the phase diagram of an ideal mixture of bosonic and fermionic atoms in thermal and chemical equilibrium with heteronuclear fermionic molecules. A cooling cycle has been identified

that exploits the mechanism of adiabatic atom-molecule interconversion. This cycle could provide a useful complement to existing cooling techniques. Its practical efficiency will be limited by effects of atom-atom and atom-molecule interactions, which lie outside the scope of the present treatment, and must be determined by experiment.

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